MATH 318 *Operations Research*
Spring 2023

Some Advice on Project 2

There are 3 stages, corresponding to the successive quarters; thus n = 1, 2 or 3.
The decision variable x_n has two values: advertise (A) or discontinue (D).
Let s_n be the level of sales above ($s_n \geq 0$) or below ($s_n \leq 0$) the breakeven point for quarter n-1.
Let a_n and b_n be the smallest and largest, respectively, possibilities for the additional sales in quarter n.
Let $f_n(s_n,x_n)$ be the maximum expected profit (in millions) from the beginning of period n onward, given state s_n and decision x_n.
Recall from calculus that the *average value* or *expected value* of a continuous function g on an interval $[a,b]$ is given by $\frac{1}{b-a}\int_a^b g(t)\,dt$.

1) Explain why each of the following is true:
   a. $s_1 = -4$
   b. $-3 \leq s_2 \leq 1$
   c. $-3 \leq s_3 \leq 5$
   d. $-4 \leq s_4 \leq 8$
2) Explain why if the decision x_n is to advertise, then we have the recursive relationship $f_n(s_n, x_n) = -30 + 5\left[s_n + \frac{a_n + b_n}{2}\right] + \frac{1}{b_n - a_n}\int_{a_n}^{b_n} f_{n+1}^*(s_n + t)\,dt$
3) The n = 3 stage. Here $-3 \leq s_3 \leq 5$. You will want to break up the [-3,5] into two ([-3,1], [1,5]) subintervals or perhaps three ([-3,-1], [-1,1], [1,5]) subintervals. The functions you will be integrating will all be linear functions of t.
4) The n = 2 stage. Here $-3 \leq s_2 \leq 1$. You will eventually want to split this interval into three subintervals: [-3, K], [K, -1] , [-1,1] where K is the value that makes $f_2(K,A) = f_2(K,D)$. You should obtain $K = \frac{-47 + 8\sqrt{10}}{9} \approx -2.411$.
5) The n = 1 stage. Here $s_1 = -20$. Show that $f_1(-20,D) = -20$ while $f_1(-20,A)$ is given by $f_1(-20, A) = -30 + 5(-4 + 3) + \frac{1}{4}\int_1^5 f_2^*(-4 + t)\,dt$. You will need to split the interval of integration [1,5] into 3 subintervals: [1,K+4], [K+4,3], [3,5]. You should arrive at a maximum expected profit of about 6.7.