Dynamic Programming IV

Class 28

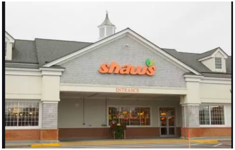
April 26, 2023

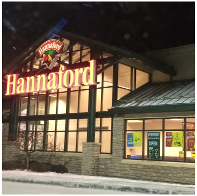
Probability Distribution for Daily Demand

Assignment 10
Team Project 2

Exam Results

Probabilistic Dynamic Programming I


shaw's
ENTRANCE


Hannaford
Hannaford


Middlebury Natural Foods Co-op

Henry Brewster decides to market the Fancy Assortment through three local outlets: Shaw's, Hannaford, and the Middlebury Co-Op.

Each day he produces 6 cases (24 boxes each) at a cost to him of \$100 a case. Each case sold at an outlet yields \$200.

Any unsold assortments are returned to the factory where he can sell them at \$50 a case as stale products the next day.

Probability Distribution for Daily Demand
(demand in cases)

		1	2	3
Store 1	Shaw's	.6	0	.4
Store 2	Hannaford's	.5	.1	.4
Store 3	Middlebury Co-Op	.4	.3	.3

Problem: **How should he allocate the 6 cases to the three outlets to maximize his expected revenue?**

Observations

- Don't give more than 3 cases to any store
- Distribute all 6

Expected Revenue Earned from Allocating x_n cases to store n (in hundreds of dollars)

x_n	Store 1 Shaw's	Store 2 Hannaford's	Store 3 Co-Op
0	$0	$0	$0
1	$2	$2	$2
2	$3.10	$3.25	$3.40
3	$4.20	$4.35	$4.35

Allocate 2 to Shaw's (Store 1)

.6 (Sell 1, Return 1) + .4 (Sell 2)

$.6(2 + 1/2) + .4(4) = 1.5 + 1.6 = 3.1$

Allocate 3 to Shaw's

.6 (Sell 1, return 2) + .4(sell 3)

$.6(2 + 1) + .4(6) = 1.8 + 2.4 = 4.2$

Allocate 2 to Hannaford's (Store 2)

.5 (Sell 1, return 1) + .5(sell 2)

$.5(2 + 1/2) + .5(4) = 3.25$

Allocate 3 to Hannaford's

.5(sell 1, return 2) + .1(sell 2, return 1) + .4(sell3)

$.5(2 + 1) + .1(4 + 1/2) + .4(6)$

$1.5 + .45 + 2.4 = 4.35$

Allocate 2 to Co-Op

.4(sell 1, return 1) + .6(sell 2)

$.4(2 + 1/2) + .6(4) = 1 + 2.4 = 3.4$

Allocate 3 to Co-Op

.4(sell 1, return 2) + .3(sell 2, return 1) + .3(sell 3)

$.4(2 + 1) + .3(4 + ?) + .3(6) = 1.2 + 1.35 + 1.8 = 4.35$

Expected Revenue Earned from Allocating x_n cases to store n (in hundreds of dollars)

x_n	Store 1 Shaw's	Store 2 Hannaford's	Store 3 Co-Op
0	$0	$0	$0
1	$2	$2	$2
2	$3.10	$3.25	$3.40
3	$4.20	$4.35	$4.35

Let $r_i(s)$ = expected revenue of giving s cases to store i. Then the recursive relationship is

$$f_3(s) = r_3(s)$$
$$f_n(s) = r_n(s) + f^*_{n+1}(x - s)$$

$$f^*_n(s) = \max_{k \leq s}[r_n(k) + f^*_{n+1}(s - k)]$$

$$f_3(s) = r_3(s)$$

$n = 3$	Co-Op					
$s \backslash x_3$	0	1	2	3	f_3^*	x_3^*
0	0	–	–	–	0	0
1	0	2	–	–	2	1
2	0	2	3.4	–	3.4	2
3	0	2	3.4	4.35	4.35	3

$$f_2^*(s) = \max_{k \le s}[r_2(k) + f_3^*(s-k)]$$

$n=2$	Hannaford					
x_2 \ s	**0**	**1**	**2**	**3**	f_2^*	x_2^*
0	0 +0 = 0	–	–	–	0	**0**
1	0 + 2 = 2	2 + 0 = 2	–	–	2	**0 or 1**
2	0+3.4=3.4	2 + 2 = 4	3.25 + 0	–	4	**1**
3	0+4.35=4.	2 +3.4= 5.4	3.25+2= 5.25	4.35 + 0 = 4.35	5.40	**1**
4	–	2 + 4.35=	3.25 + 3.4=6.65	4.35 + 2 = 6.35	6.65	**2**
5	–	–	3.25 + 4.35=7.60	4.35 + 3.4 =7.75	7.75	**3**
6	–	–	–	4.35+4.35 =8.70	8.70	**3**

$$f_1^*(s) = \max_{k \leq s}[r_1(k) + f_2^*(s-k)]$$

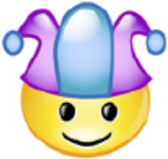
$n = 1$	Shaw's					
x_1 \ s	**0**	**1**	**2**	**3**	f_1^*	x_1^*
6	**0 + 8.70 = 8.70**	**2 + 7.75 = 9. 75**	**3.10 +6.65 = 9.75**	**4.2 + 5.40 = 9.60**	**9.75**	**1 or 2**

Solutions

Store	Number of Cases	Number of Cases
Shaw's	1	2
Hannaford	3	2
Co-Op	2	2

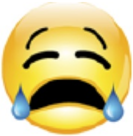
Expected Value of Revenue: $975

Expected Value of Revenue: \$975



Best Possible Outcome

Sell all 6 cases → \$1200



Worst Possible Outcome

Sell 1 at each store

3 sales, 3 returns

Revenue $= 3 \cdot \$200 + 3 \cdot \$50 = \$750$

Probabilistic Dynamic Programming II

Choose The Ideal Mate

Choosing The Ideal Mate

Choosing The Ideal Mate

N candidates each with values 0 - 1000, uniformly distributed.

Uniform Distribution: For all a and b with $0 \leq a < b \leq 1000$, the probability that a value lies in the interval $[a, b]$ is proportional to the length of the interval. In this case, the probability would be $\frac{b-a}{1000}$.

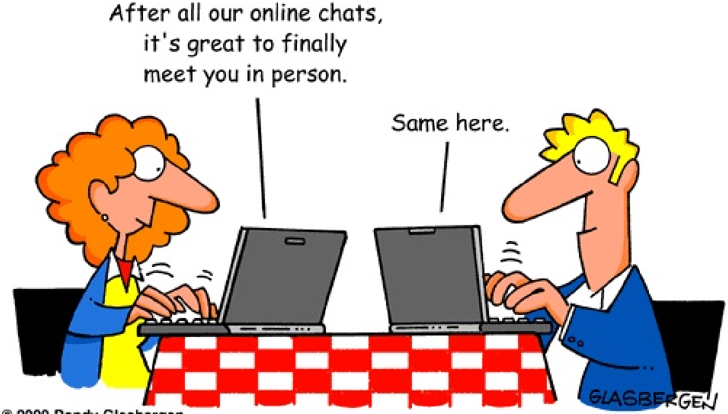
If we choose at random a number in an interval $[a, b]$, its expected value is the average of a and b.

Candidates are chosen at random and presented to you one at a time.

You either Accept or Reject and move on to the next candidate.

Our Question: How should you proceed if you wish to maximize the expected value of the candidate accepted?

[Different Question: What is the best policy if you wish to maximize the probability of picking the best candidate among the N?]



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Stages : candidates
x_n: decision variable" accept or reject.
s: value of current candidate
Stage N: Nth candidate has value uniformly distributed on [0,1000] so the Expected Value is 500.
Stage $N-1$: Consider $N-1$st Candidate:

Accept if $s > 500$; that is s is in the interval [500,1000]
Expected Value is $\frac{500+1000}{2} = 750$.
Reject is $s \leq 500$ in which case the Expected Value is 500
Thus $\mathbf{EV} = \frac{1}{2}(750) + \frac{1}{2}(500) = \frac{1}{2}(750 + 500) = 625$

Stage $N-2$: Consider $N-2$nd Candidate:

Accept if $s > 625$; that is, s is in the interval [625,1000]
Expected Value is $\frac{625+1000}{2} = 812.50$
Reject is $s \leq 625$ in which case the Expected Value is 625
Thus $\mathbf{EV} = (.625)(625) + (1 - .625)(812.50) = 695.3$

Stage $N-3$: Consider $N-3$rd Candidate:

Accept if $s > 695.3$; that is, s is in the interval [695.3,1000]
Expected Value is $\frac{695.3+1000}{2} = 847.5$
Reject is $s \leq 695.3$ in which case the Expected Value is 695.3
Thus $\mathbf{EV} = (.6953)(695.3) + (1 - .6953)(847.50) = 741.5$

Stage $N-3$: Consider $N-3$rd Candidate:

Accept if $s > 695.3$; that is, s is in the interval [695.3,1000]
Expected Value is $\frac{695.3+1000}{2} = 847.5$
Reject is $s \leq 695.3$ in which case the Expected Value is 695.3
Thus $\mathbf{EV} = (.6953)(695.3) + (1 - .6953)(847.50) = 741.5$

General Case

Accept if $s > x$; that is, s is in the interval [x,1000]
Expected Value is $\frac{x+1000}{2}$
Reject is $s \leq x$ in which case the Expected Value is x
Thus $\mathbf{EV} = \frac{x}{1000}x + (1 - \frac{x}{1000})(\frac{x+1000}{2})$
$\mathbf{EV} = \frac{x^2}{1000} + (\frac{1000-x}{1000})(\frac{x+1000}{2})$

$$\textbf{EV} = \frac{x^2}{1000} + \left(\frac{1000-x}{1000}\right)\left(\frac{x+1000}{2}\right)$$

$$\textbf{EV} = \frac{2x^2}{2000} + \frac{1000^2-x^2}{2000} = \frac{x^2+1000^2}{2000}$$

Improvement: $\frac{x^2+1000^2}{2000} - x = \frac{x^2-2000x+1000^2}{2000} = \frac{(x-1000)^2}{2000}$

Sequence: $x_1 = 500; x_n = \frac{x_n^2 + 1000^2}{2000}$

500

625

695

742

77

800

820

836

850

861

...

The sequence has a limit L which satisfies $L = \frac{L^2 + 1000^2}{2000}$

$L^2 - 2000L + 1000^2 = 0 \to (L - 1000)^2 = 0 \to L = 1000$

What is the Incremental Improvement in Looking at One More Candidate?

$$\frac{(x-1000)^2}{2000}$$

$$x = 500 : \text{improvement is } \frac{(500-1000)^2}{2000} = \frac{250000}{2000} = 125$$

$$x = 850 : \text{improvement is } \frac{(850-1000)^2}{2000} = \frac{22500}{2000} = 11.25$$

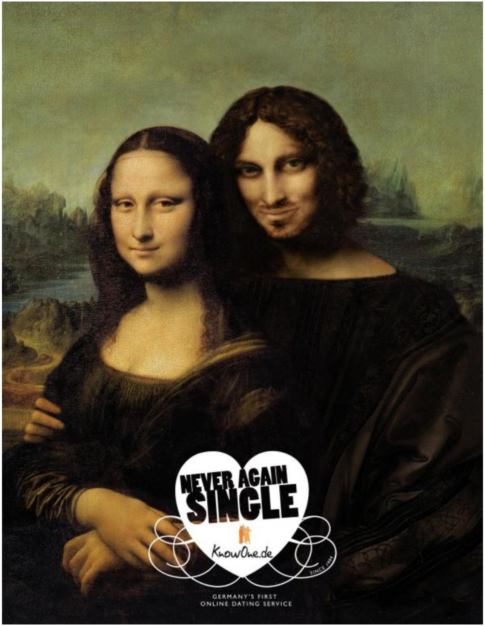
When is the Incremental Improvement less than 5?

$$\frac{(x-1000)^2}{2000} < 5$$

$$(x-1000)^2 > 5(2000) = 10000 = 10^4$$

$$|x-1000| < 10^2$$
$$|x-1000| < 100$$

AS SOON AS $x > 900$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