

# The Used Car Problem

You've just purchased a used car for \$12,000 and want to determine the best keep/trade-in strategy over the half dozen years.

Age in Car (years)	Annual Maintenance Cost	Trade-In Price At End of Year	Replacement Cost
0	\$2,000	\$7,000	\$12,000
1	\$4,000	\$6,000	\$12,000
2	\$5,000	\$2,000	\$12,000
3	\$9,000	\$1,000	\$12,000
4	\$12,000	\$0	\$12,000

Goal: Minimize Net Costs:

Maintenance Cost + Purchasing Cost - Trade-In Money Received

## Formulate as a Network Problem

Node  $i$  = Beginning of year  $i$ .

For  $i < j$ , an arc  $(ij)$  corresponds to purchasing a car at the beginning of year  $i$  and keeping it until the beginning of year  $j$ .

Let  $c_{ij}$  = cost of using arc  $ij$ .

Now  $c_{ij}$  = cost of purchasing car at start of year  $i$  + Maintenance cost incurred in years  $i, i + 1, i + 2, \dots, j - 1$  - Trade-in at beginning of year  $j$ .

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Some Examples (measured in thousands of dollars)

$$c_{12} = 12 + 2 - 7 = 7$$

$$c_{13} = 12 + (2 + 4) - 6 = 12$$

$$c_{14} = 12 + (2 + 4 + 5) - 2 = 21$$

$$c_{15} = 12 + (2 + 4 + 5 + 9) - 1 = 31$$

$$c_{16} = 12 + (2 + 4 + 5 + 9 + 12) - 0 = 44$$

Note that  $c_{13} = c_{24} = c_{35} = c_{46}$  and, in general  $c_{ij} = c_{i+k,j-k}$