Introduction to Operations Research

Class 2

February 15, 2023

Handouts

Tables for Fromage and Transportation Problem
Linear Programming I (pdf file online)

Linear Algebra

Review Linear Algebra:

Reduce Matrix to Row Echelon Form
Matrix Multiplication
Transpose of a Matrix
Dot Product of Vectors
Elementary Properties of Linear Functions
Concept of a Basis for a Set of Vectors in R^n.

Review

OPERATIONS RESEARCH IS
THE DEVELOPMENT AND APPLICATION
OF SCIENTIFIC METHODS
TO PROVIDE A QUANTITATIVE BASIS
FOR DECISION MAKING
IN AN ENVIRONMENT CHARACTERIZED
BY COMPLEXITY AND UNCERTAINTY

Review: Characteristics of Operations Research

Characteristics of Operations Research Analysis Today

- Primary Focus on Decision Making
  Princpal results of the analysis must have direct and unambiguous implication for action.
- An Appraisal Resting on Economic Effectiveness Criteria
  Comparison of various feasible actions must be based on measurable values
- Reliance on a Formal Mathematical Model
- Dependence on a Computer
  Complexity of model
  Volume of data
  Magnitude of required computations


Operations Research
Is Max/Min Problems
For GrownUps

Review

Questions from Last Time?

Dynamic Programming: Chapter 10 of Hillier and Lieberman

Dynamic Programming is relatively new technique.

Classical Calculus Optimization ($f'(x) = 0$) is much older, but still fruitful

Example: Inventory Model in CBL

Example: Farmer Brown Problem

A Calculus I Problem

Farmer Brown has 200 feet of fencing and wants to use it to enclose a rectangular field. What dimensions maximize area?

Formulating the Model

Let x be the length in feet and y be the width in feet of the rectangle.

The Problem is Maximize $M = xy$

x, y are the *Decision Variables*

xy is the *Objective Function*

But there are constraints on the variables x and y

Farmer can't use more than 200 feet of fencing.

$$2x + 2y \leq 200$$

Dimensions of the field must be non-negative

$$x \geq 0, y \geq 0$$

The Model

$$\text{Maximize } M = xy$$

subject to the constraints

$$2x + 2y \leq 200$$
$$x \geq 0$$
$$y \geq 0$$

Our Central Problem

The Fromage Cheese Company Problem

Assortment	Cheddar	Swiss	Brie	Price	Number of Packages
Fancy	30	10	4	$4.50	x
Deluxe	12	8	8	$4.00	y
Available	6000	2600	2000		

Note: All quantities of cheeses are in ounces
Question: How many packages of each mixture should we make that will maximize revenue but not exceed our cheese supplies?

$$\begin{gathered}\text{Problem: Maximize } M = 4.5x + 4y \\ \text{subject to constraints} \\ 30x + 12y \leq 6000 \text{ (Cheddar)} \\ 10x + 8y \leq 2600 \text{ (Swiss)} \\ 4x + 8y \leq 2000 \text{ (Brie)} \\ x, y \geq 0\end{gathered}$$

Dual of the Fromage Problem

Suppose you want to buy all of the cheese for your own purposes. How much should you offer?
Let c = dollars per ounce of Cheddar
Let s = dollars per ounce of Swiss
Let b = dollars per ounce of Brie

Then you want to minimize $6000c + 2600s + 2000b$
subject to
$30c + 10s + 4b \geq 4.5$
$12c + 8s + 8b \geq 4$
$c, s, b \geq 0$
This problem is called the **DUAL** of the Fromage Problem which is called the **PRIMAL**

A Relationship Between Primal and Dual

Primal	**Dual**
Maximize $M = 4.5x + 4y$	Minimize $6000c + 2600s + 2000b$
subject to	subject to
$30x + 12y \leq 6000$	$30c + 10s + 4b \geq 4.5$
$10x + 8y \leq 2600$	$12c + 8s + 8b \geq 4$
$4x + 8y \leq 2000$	
$x, y \geq 0$	$c, s, b \geq 0$

$$\left(\begin{array}{cc|c} 30 & 12 & 6000 \\ 10 & 8 & 2600 \\ 4 & 8 & 2000 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 30 & 10 & 4 & 4.5 \\ 12 & 8 & 8 & 4 \end{array}\right)$$

Transportation Problem

Transportation Problem

3 factories: Seattle, San Francisco, Minneapolis
4 warehouses (WH): Sacramento, Salt Lake City, Rapid City SD, Albuquerque

	WH 1	WH 2	WH 3	WH 4	Supply
Factory 1	464	513	654	867	75
Factory 2	352	416	690	791	125
Factory 3	995	682	388	685	100
Demand	80	65	70	85	300

Supply and demand measured in truckloads.
Matrix elements: shipping costs in dollars per truckload

Problem: Minimize total shipping costs while meeting demand and not exceeding supply.

Decision Variables: x_{ij} = number of truckloads to ship from factory i to warehouse j.

$$\text{Minimize } m = 464x_{11} + 513x_{12} + \ldots + 685x_{34}$$

subject to supply constraints

$$\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} &\leq 75 \\ x_{21} + x_{22} + x_{23} + x_{24} &\leq 125 \\ x_{31} + x_{32} + x_{33} + x_{34} &\leq 100 \end{aligned}$$

and demand constraints

$$\begin{aligned} x_{11} + x_{21} + x_{31} &\geq 80 \\ x_{12} + x_{22} + x_{32} &\geq 65 \\ x_{13} + x_{23} + x_{33} &\geq 70 \\ x_{14} + x_{24} + x_{34} &\geq 85 \end{aligned}$$

and nonnegatiivity constraints

$$\text{each } x_{ij} \geq 0$$

The Linear Programming Problem

Each of the last 3 problems (Fromage, its Dual, Transportation) is an example of what is called a **Linear Programming Problem:**

Optimize a LINEAR function subject to a finite number of LINEAR constraints (equations or inequalities)

Some Linear Programming Applications

- Product Mix
- Transportation Problems
- Assignment Problems
- Diet Problems
- Media Selection
- Blending Problems
- Portfolio Selection
- Project Management Problems
- Market Research Problems