Introduction to Operations Research

Class 1

February 13, 2023

Announcements: Grader Jobs

Homework Grader Positions Are Available

See Elizabeth Kervick for Details

https://forms.middlebury.edu/academics/math/grader-application

Handouts

Course Packet

- Course Description
- Topics
- Schedule
- Assignment 1
- Cooper, Bhat and LeBlanc Reading

Questionnaire

Online

- Mathematics and Operations Research in Industry
- Subject Classification Scheme
- Other interesting links

Other Names for OR

Operations Research
Operational Research
Management Science
Systems Engineering
Decision Analysis
Engineering Management, Information and Systems
Descriptive, Predictive, and Prescriptive Analytics

Definition of OR

OPERATIONS RESEARCH IS
THE DEVELOPMENT AND APPLICATION
OF SCIENTIFIC METHODS
TO PROVIDE A QUANTITATIVE BASIS
FOR DECISION MAKING
IN AN ENVIRONMENT CHARACTERIZED
BY COMPLEXITY AND UNCERTAINTY

Tentative List of Topics

Two Particular Problems

Decision Making Under Uncertainty via Dynamic Programming

Inventory Theory via Classic Calculus

Origins, Nature and Impact of Operations Research

Overview of the OR Modeling Approach

Linear Programming

The Simplex Method of Solving LP Problems

Duality Theory

The Theory of the Simplex Method

Sensitivity Analysis

Transportation and Assignment Problems

Network Optimization models

Integer Programming: A Brief Look

Nonlinear Programming: A Brief Look

Game Theory

Example: 2 Job Interview Problems

Job Interview Problem 1

After several preliminary telephone interviews, email exchanges, and a campus visit by recruiters, you are invited by 3 separate potential employers for a fine on-site job interview.

Quality of Job	Probability of Having It Offered		
	Boston	New York	San Francisco
Terrific	.1	.2	.2
Good	.6	.3	.4
Fair	.3	.5	.4

Is the San Francisco possibility superior to New York?
Is New York better than Boston?
Need to attach some measure of value to Terrific, Good and Fair.

Job Interview Problem 1

Suppose we value by **SALARY**

Terrific: \$120K Good: \$ 100K Fair: \$80K

Quality of Job *Probability of Having It Offered*

	Boston	New York	San Francisco
Terrific(\$120K)	.1	.2	.2
Good(\$100K)	.6	.3	.4
Fair(\$80K)	.3	.5	.4

Use idea of **EXPECTED VALUE**

EV = weighted sum of each outcome weighted by its probability

(more on Expected Value in a few minutes)

Boston: $.1(120) + .6(100) + .3(80) = 12 + 60 + 24 =$ \$96K

New York $.2(120) + .3(100) + .5(80) = 24 + 30 + 40 =$ \$94K

San Francisco $.2(120) + .4(100) + .4(80) = 24 + 40 + 32 =$ \$96K

"My short-term goal is to bluff my way through this job interview. My long-term goal is to invent a time machine so I can come back and change everything I've said so far."

Job Interview Problem 2

You must accept or reject the proposed job at the time it is offered. If you turn down an offer, there is no second chance to come back later after interviewing at the other companies.
What should your strategy be?

What if first job offer is TERRIFIC?
Take it. You cannot do better and you might do worse.

What if first job offer is FAIR?
Reject it. You cannot do worse and you might do better at a later interview.

What if first job offer is GOOD?
Unclear. Accepting means you give up Terrific possibility. Reject means you might get stuck with Fair job.

To determine an **OPTIMAL STRATEGY** for you, we need to know two things:

1. What are the relative values of the 3 types of jobs to you?
2. What are you trying to optimize?

To Answer First Question:

Suppose you assign the following values:

Likelihood	Type of Job	Value
.2	Terrific	3
.5	Good	2
.3	Fair	1

To Answer Second Question:

Consider situation of a single interview

Expected Value

Suppose an experiment has possible numerical outcomes $a_1, a_2, ..., a_k$ with corresponding probabilities $p_1, p_2, ..., p_k$.
Then the Expected Value EV $= a_i p_1 + a_2 p_2 + ... + a_k p_k$

Likelihood	Type of Job	Value
.2	Terrific	3
.5	Good	2
.3	Fair	1

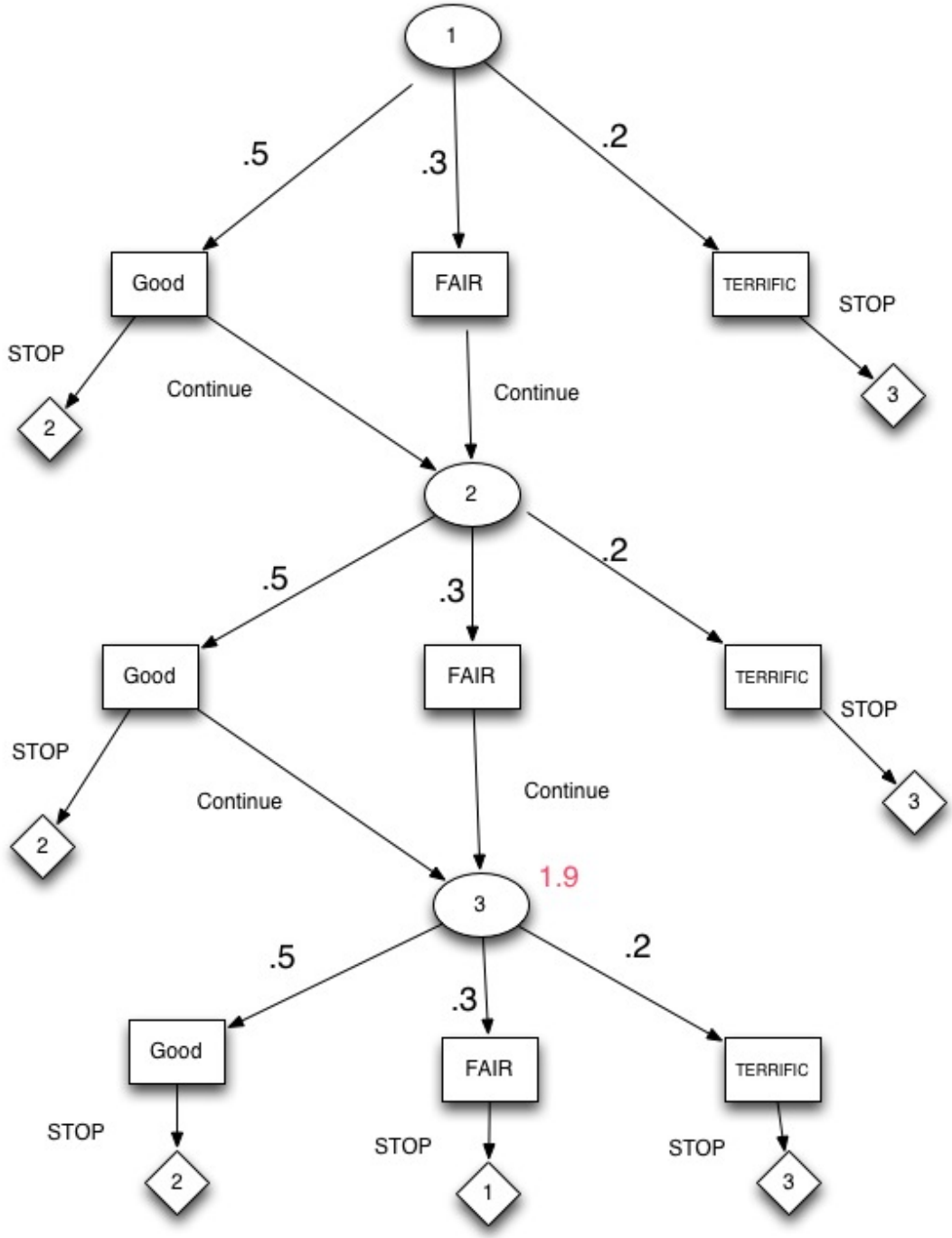
In our case
EV $= 3(.2) + 2(.5) + 1(.3) = .6 + 1 + .3 = 1.9.$

Interpretation of Expected Value

Imagine 100 identical candidates apply.
Then we expect 20 Terrifics, 50 Goods and 30 Fairs.
Total Value is $20 \times 3 + 50 \times +30 \times 1$
Average Value is
$\frac{20\times3+50\times+30\times1}{100} = 3(.2) + 2(.5) + 1(.3) = .6 + 1 + .3 = 1.9.$

Decision Trees and Dynamic Programming

We are now ready to determine the **optimal strategy**, the strategy that *maximizes* the expected value of the job you will receive.
First draw a **Decision Tree**
Circle: Job Interviews
Branches from Circles: chance events and their probabilities
Boxes: Where a decision has to be made
Numbers at end of boxes: Relative value of stopping at that point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FAIR
TERRIFIC
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STOP
Continue
Continue
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3
.5
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Good
FAIR
TERRIFIC
STOP
STOP
STOP
2
1
3

Dynamic Programming and Backward Induction

We can find the optimal decision strategy by the technique called **DYNAMIC PROGRAMMING** using a process known as **Backward Induction**

Suppose you arrive at the third interview. The expected value of the job is 1.9

Consider now what to do should you be at the second interview and you are offered the Good job

STOP: Get 2 in value

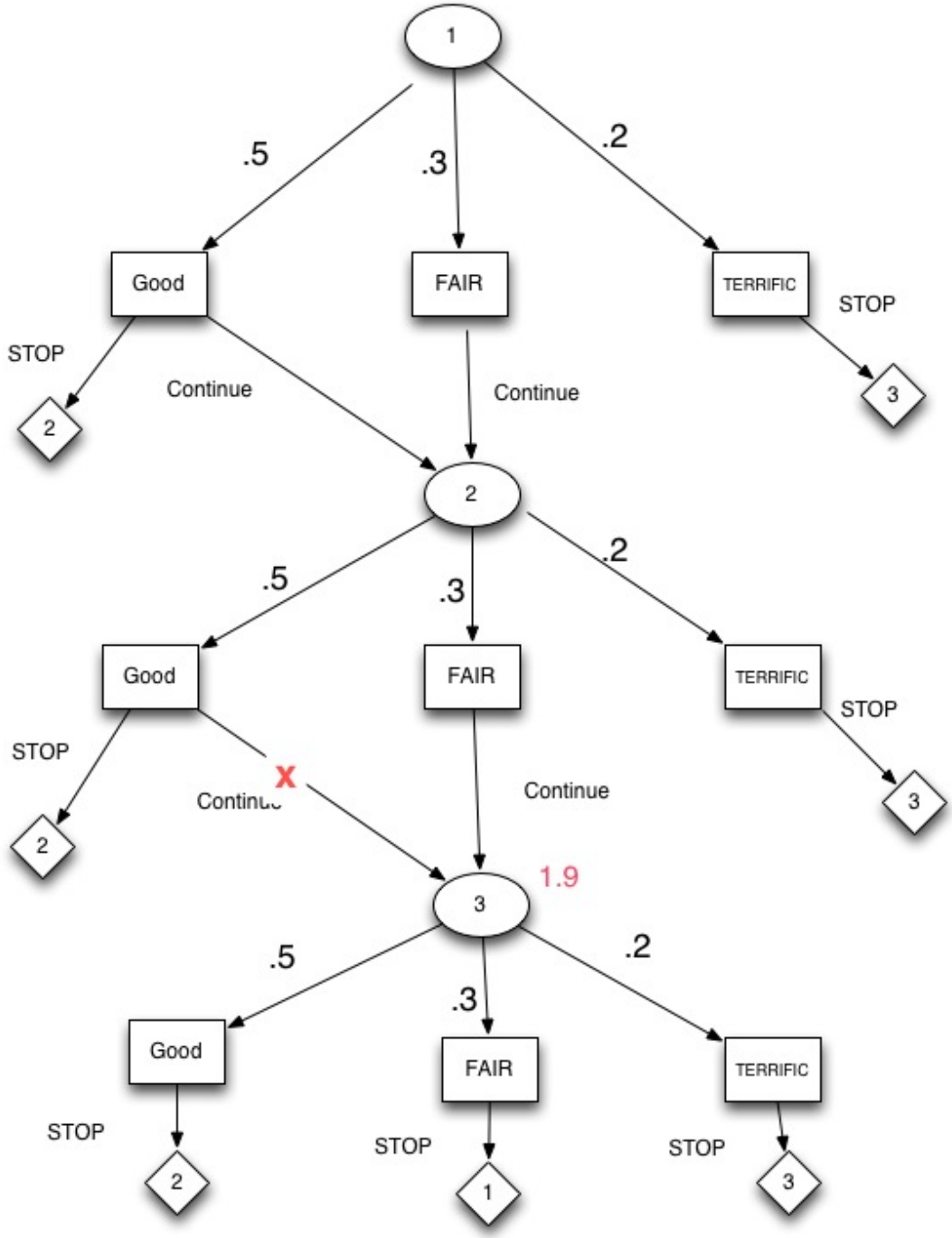
CONTINUE: Expect 1.9 in value.

Put x on "continue" after G offer.

Value of Interview 2 is: $3(.2) + (1.9)(.3) + (2)(.5)$

$= .6 + .57 + 1 = 2.17.$

Incremental Value of Interview 2 is 2.17 - 1.9 = .27.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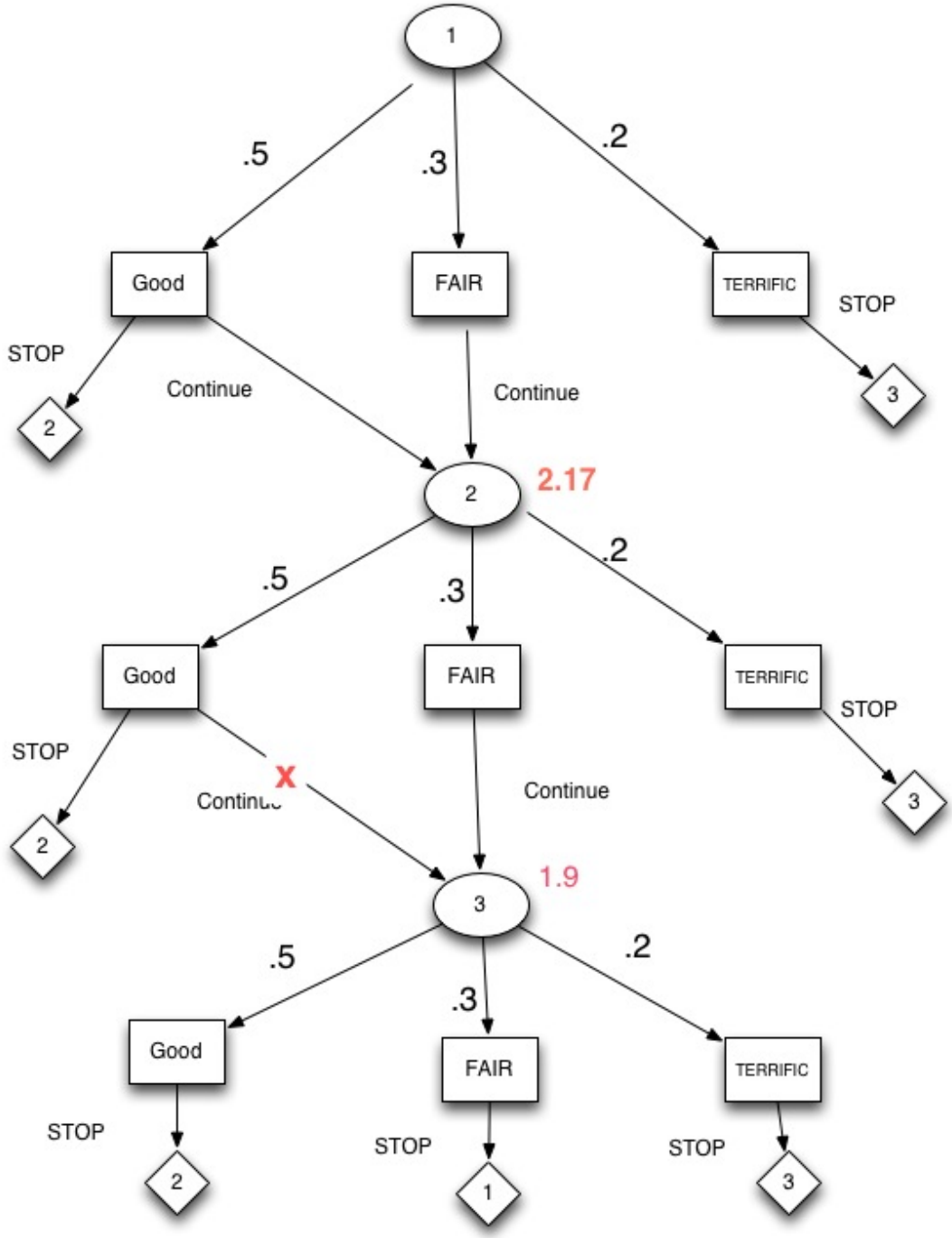
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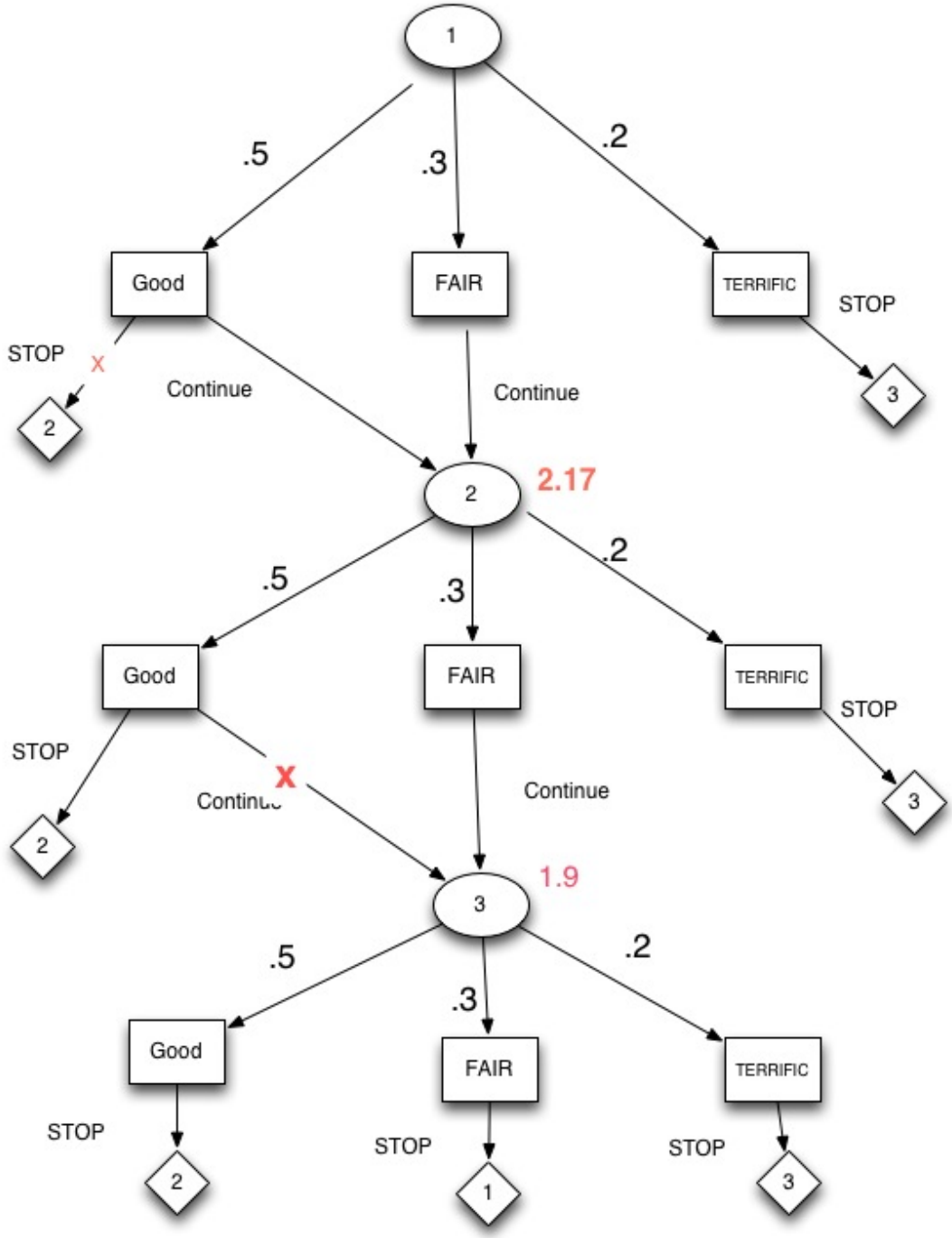
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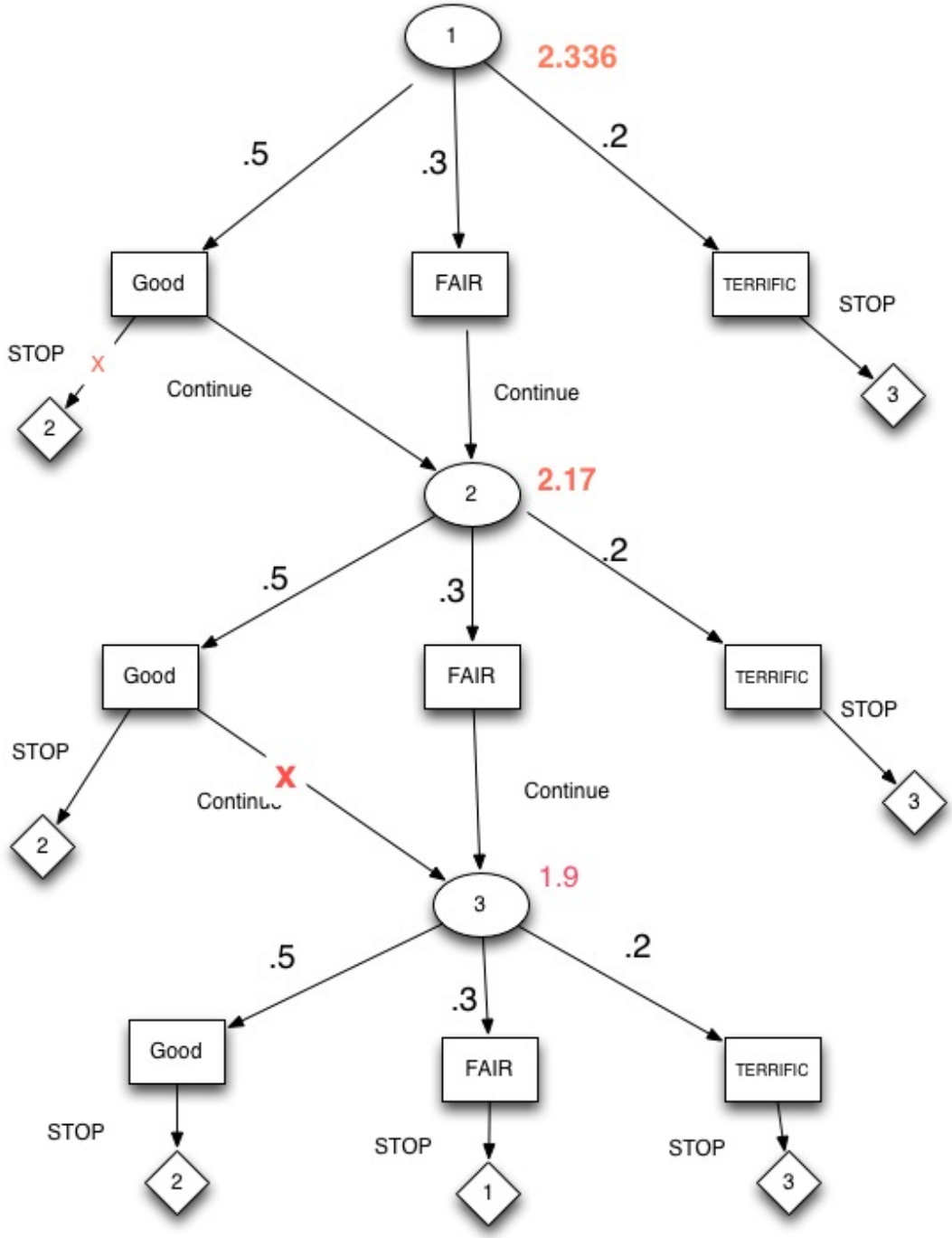
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Continue
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3
1.9
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.3
.2
Good
FAIR
TERRIFIC
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STOP
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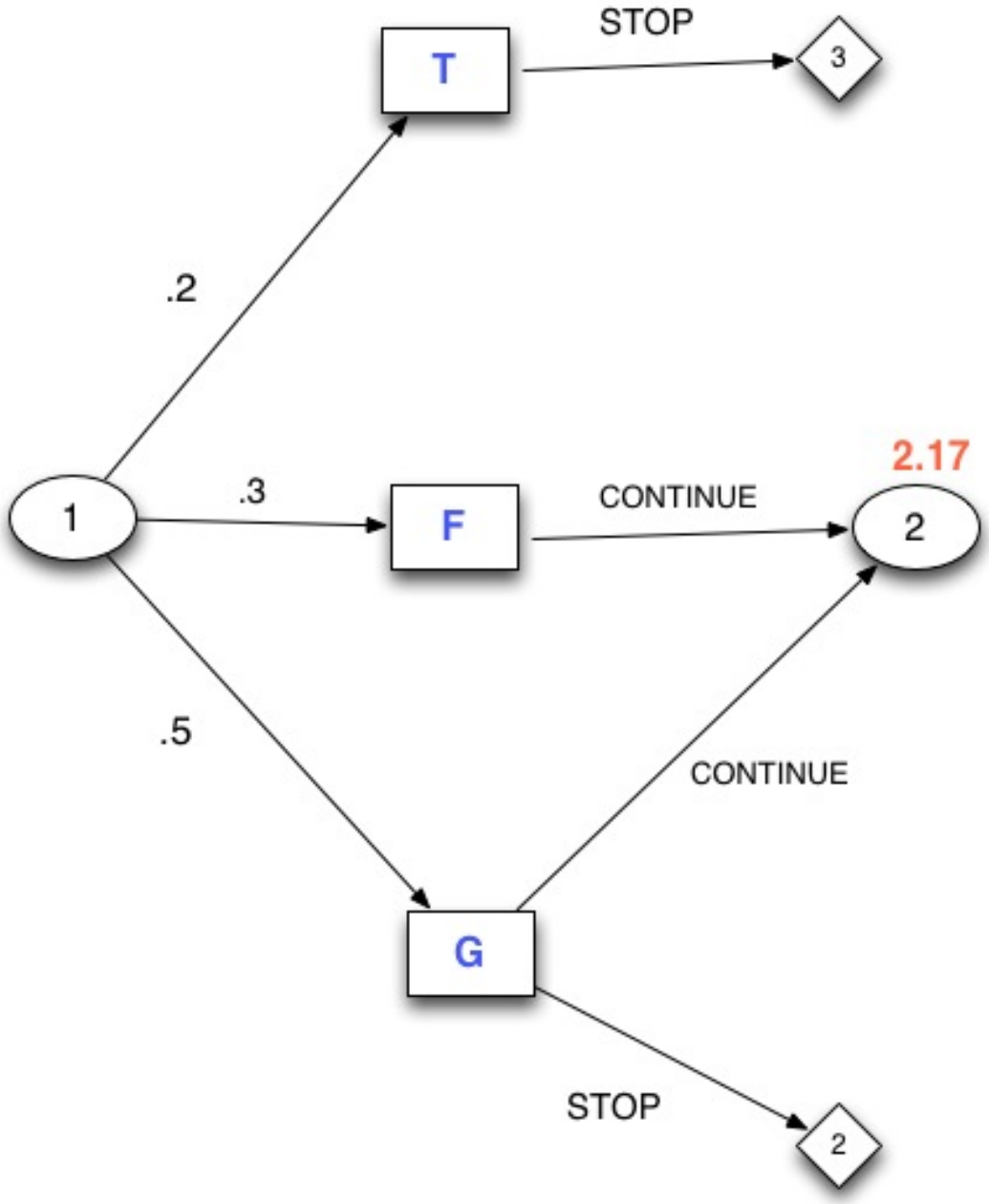
3

Dynamic Programming and Backward Induction

Now look at First Interview:
STOP at Good Offer has value 2.
Continue to Second Interview has value 2.17.
Therefore, you should continue interviewing.


STOP
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.2
2.17
1
.3
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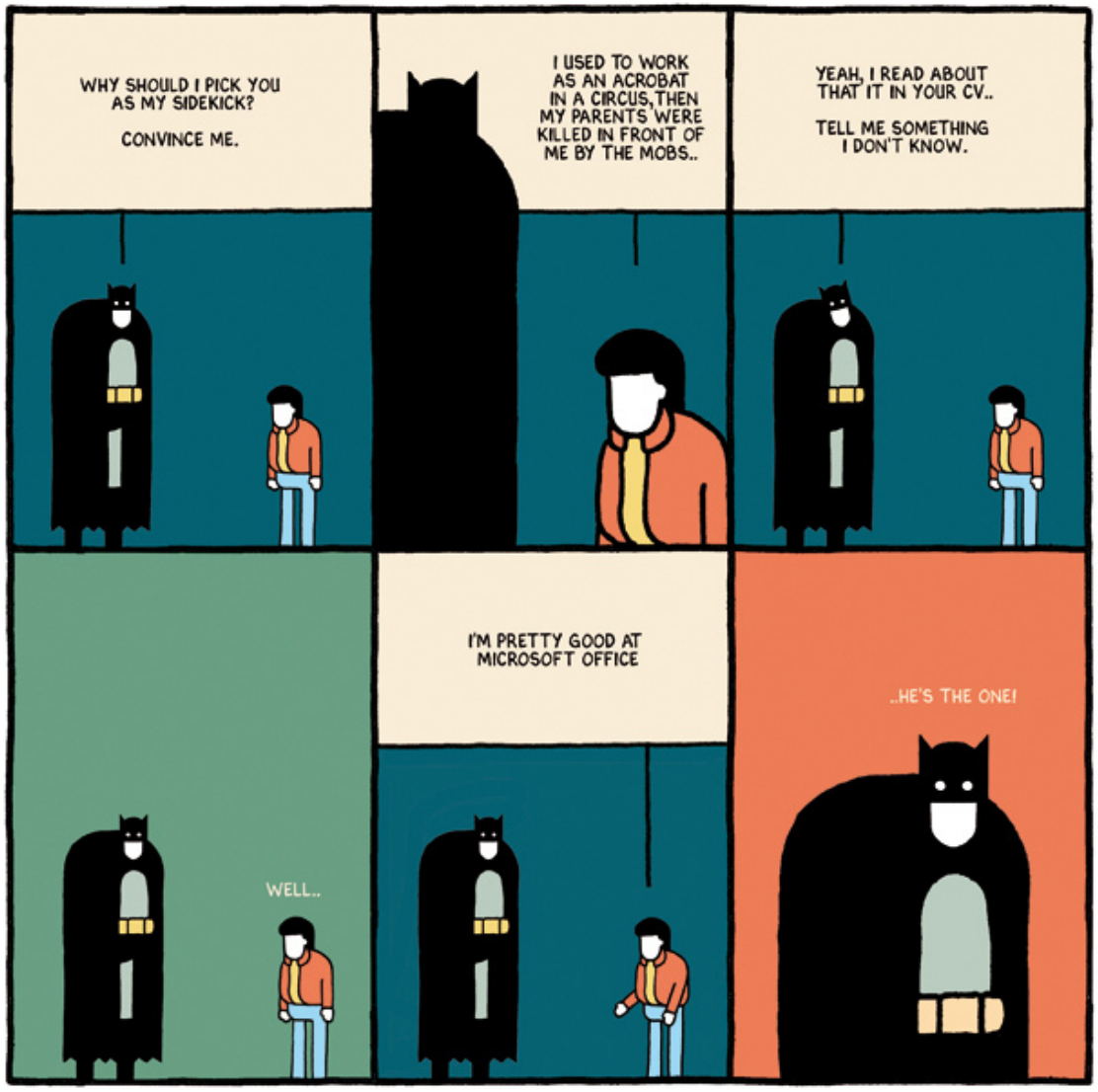
CONTINUE
2
.5
CONTINUE
G
STOP
2

Statement of Optimal Policy

The conclusion of your analysis should be a clear statement of a recommended optimal policy:

At First Interview: Accept offer only if it is TERRIFIC; otherwise, continue to the second interview.
At Second Interview (if it occurs): Accept if TERRIFIC or GOOD offer;otherwise continue to the third interview.
At Third Interview(if it occurs): Accept any offer.
The Expected Value Under This Optimal Policy is:
$3(.2) + (2.17)(.3) + (2.17)(.5) = 2.336$.
Incremental value of 3 interviews over 1: 2.336 - 1.9 = .436.

THE JOB INTERVIEW



Extensions

1. n interviews
2. Unspecified number of interviews
3. Cost associated with each interview
4. Different probabilites at each interview
5. Waiting time to make decision
6. Hiring Problem: Suppose you want to hire more than one person
7. Application to jury selection

Review: Characteristics of Operations Research

Characteristics of Operations Research Analysis Today

- **Primary Focus on Decision Making**
  Princpal results of the analysis must have direct and unambiguous implication for action.
- **An Appraisal Resting on Economic Effectiveness Criteria**
  Comparison of various feasible actions must be based on measurable values
- **Reliance on a Formal Mathematical Model**
- **Dependence on a Computer**
  Complexity of model
  Volume of data
  Magnitude of required computations