The Penultimate Class

Class 35

May 12, 2023

Handouts

Notes on Assignment 11

Vogel's Method

Notes on Project 2

Final Examination
Thursday May 18
9 AM - Noon

Course Response Forms
Monday
Bring Laptop/Smart Phone

An Unbalanced Transportation Problem

Vogel's Method and Unbalanced Transportation Problem

	W1	W2	W3	Supply
F1	**2**	**1**	**3**	200
F2	**2**	**2**	**4**	100
F3	**1**	**4**	**3**	400
Demand	150	120	300	

Table 1: Original Data. F1, F2, F3 are Factories while W1, W2, W3 are the warehouses.
Red Numbers are shipping costs per truckload.
Here Supply (700) exceeds Demand (570) by 130

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NORTHWEST CORNER RULE

	W1	W2	W3	W4	Supply
F1	**2** **150**	**1** **50**	**3**	**0**	200
F2	**2**	**2** **70**	**4** **30**	**0**	100
F3	**1**	**4**	**3** **270**	**0** **130**	400
Demand	150	120	300	130	

TOTAL COST: 1420

Northwest Corner Rule produces a basic feasible solution with objective function value 1420.

Vogel's Method produces one with value 1170.

MATH 318 Operations Research

Vogel's Method and Unbalanced Transportation Problem

	W1	W2	W3	Supply
F1	2	1	3	200
F2	2	2	4	100
F3	1	4	3	400
Demand	150	120	300	

Table 1: Original Data. F1, F2, F3 are Factories while W1, W2, W3 are the warehouses.
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	W1	W2	W3	W4	Supply
F1	2	1	3	0	200
F2	2	2	4	0	100
F3	1	4	3	0	400
Demand	150	120	300	130	

Table 2: We create an artificial warehouse (W4) with demand =130 so we now have a Balanced Transportation Problem.
We will illustrate Vogel's Method of obtaining initial basic feasible solution

	W1	W2	W3	W4	Supply	**Penalty**
F1	2	1	3	0	200	1
F2	2	2	4	0	100	2
F3	1	4	3	0	400	1
Demand	150	120	300	130		
Penalty	1	1	1	0		

Table 3: For each row and column, the "penalty" is the difference between the smallest and second smallest cost in that row or column. Pick the row or column with the smallest penalty. Choose arbitrarily if there is a tie.

	W1	W2	W3	W4	Supply	Penalty
F1	2	1	3	0	200	1
F2	2	2	4	0 **100**	~~100~~ 0	2
F3	1	4	3	0	400	1
Demand	150	120	300	~~130~~ 30		
Penalty	1	1	1	0		

Table 4: Pick cell in chosen row or column which has smallest cost. Make that decision variable basic, assigning value equal to the smaller of Supply and Demand for that cell. In this case, we chose to make F2W4 basic, with value = min(100,130) = 100 we have used all the supply from F2 so we will ignore that row from now on.

	W1	W2	W3	W4	Supply	Penalty
F1	2	1	3	0 **30**	~~200~~ 170	1
F2	2	2	4	0 **100**	~~100~~ 0	2
F3	1	4	3	0	400	1
Demand	150	120	300	~~130~~ ~~30~~ 0		
Penalty	1	1	1	0		

Table 5: Smallest remaining penalty is still in W4 column. We make F1W4 basic

	W1	W2	W3	W4	Supply	Penalty
F1	2	1 **120**	3	0 **30**	~~170~~ 50	1
F2	2	2	4	0 **100**	0	2
F3	1	4	3	0	400	1
Demand	150	~~120~~ 0	300	0		
Penalty	1	1	1	0		

Table 6: Now the W4 demand has been met so we will ignore this column. The smallest remaining penalty is 1. We can choose any remaining row or column with penalty 1. We chose F1W2 and made it basic. This meets W2's demand so ignore that column later.

	W1	W2	W3	W4	Supply	Penalty
F1	2	1 **120**	3	0 **30**	50	1
F2	2	2	4	0 **100**	0	2
F3	1 **150**	4	3	0	~~400~~ 250	1
Demand	~~150~~ 0	0	300	0		
Penalty	1	1	1	0		

Table 7: Make F3W1 Basic

	W1	W2	W3	W4	Supply	Penalty
F1	2	1 **120**	3	0 **30**	50	1
F2	2	2	4	0 **100**	0	2
F3	1 **150**	4	3 **250**	0	~~250~~ 0	1
Demand	0	0	~~300~~ 50	0		
Penalty	1	1	1	0		

Table 8: Make F3W3 Basic

	W1	W2	W3	W4	Supply	Penalty
F1	2	1 **120**	3 **50**	0 **30**	~~50~~ 0	1
F2	2	2	4	0 **100**	0	2
F3	1 **150**	4	3 **250**	0	0	1
Demand	0	0	~~50~~ 0	0		
Penalty	1	1	1	0		

Table 8: Make F1W3 Basic

	W1	W2	W3	W4	Supply
F1	2	1 **120**	3 **50**	0 **30**	200
F2	2	2	4	0 **100**	100
F3	1 **150**	4	3 **250**	0	400
Demand	150	120	300	150	

Table 9: We now have an initial basic feasible solution. We've put back the original supplies and demands for convenience. Now test for optimality.

u1 + v2 = 1 u1 + v3 = 3 u1 + v4 = 0 u2 + v4 = 0 u3 + v1 = 1 u3 + v3 = 3	u1 = 0 u2 = 0 u3 = 0	v1 =1 v2 = 1 v3 = 3 v4 = 0	t11 = 2 – 0 – 1 = 1 t21 = 2 - 0 – 1 =1 t22 = 2 – 0 – 1 = 1 t23 = 4 – 0 - 3 = 1 t32 = 4 – 3 – 1 = 0 t34 = 0 – 0 – 0 = 0

Table 10: Test for Optimality. All the tij are greater than or equal to 0 so we have an optimal solution. If a $t_{ij} = 0$, that indicates that we have multiple optimal solutions. Here we can obtain another optimal solution by letting F3W2 or F3W4 basic.

	W1	W2	W3	W4	Supply
F1	2	1 **120**	3 **50**$^+$	0 **30**$^-$	200
F2	2	2	4	0 **100**	100
F3	1 **150**	4	3 **250**$^-$	0 **y**$^+$	400
Demand	150	120	300	150	

Table 11: Making F3W4 basic. Least negative square is 30 so we add 30 to F3W4 and F1W3, subtract 30 from F1W4 and F3W3. Note that F1W4 will leave the basis.

	W1	W2	W3	W4	Supply
F1	2	1 **120**	3 **~~50~~**$^+$ **80**	0 **~~30~~**$^-$	200
F2	2	2	4	0 **100**	100
F3	1 **150**	4	3 **~~250~~**$^-$ **220**	0 **30**	400

Table 12: Carry out the steps indicated by comments under Table 11.

	W1	W2	W3	W4	Supply
F1	2	1 **120**	3 **80**	0	200
F2	2	2	4	0 **100**	100
F3	1 **150**	4	3 **220**	0 **30**	400
Demand	150	120	300	150	

Table 13: Result of the Iteration. Now test for optimality

u1 + v2 = 1	u1 = 0	v1 =1	t11 = 2 – 0 – 1 = 1
u1 + v3 = 3	u2 = 0	v2 = 1	t14 = 0 – 0 – 0 = 0
u2 + v4 = 0	u3 = 0	v3 = 3	t21 = 2 - 0 – 1 = 1
u3 + v1 = 1		v4 = 0	t22 = 2 – 0 – 1 = 1
u3 + v3 = 3			t23 = 4 – 0 -3 = 1
u3 + v4 = 0			t32 = 4 - 0 – 1 = 3
			OPTIMAL!

Table 14: Carry out optimality test. All the t_{ij}'s are ≥ 0 so we have reached an optimal solution.

Some Important OR Topics Yet To Be Explored:

Integer Programming
Quadratic and Other Nonlinear Programming
Combinatorial Programming
Assignment Problems
Inventory Theory
Queues
Decision Theory
Markov Decision Processes
Simulation
Supply Chain Management