MATH 318: Operations Research   Spring 2023

Assignment 4

Due: Monday, March 13

READ: Hillier & Lieberman, Chapter 4, Sections 1-6, pp. 89 - 129.

PROBLEMS: **4.3-8; 4.4-4af**; **4.5-2a-e** , and **4.6-7**

• A designation of I next to a problem in the text tells you to use your interactive IOR routine.
• 0n 4.6-7, the idea for (a) and (c) is for you to go through the formulation of the initial tableaux once by hand for practice. The rest of the problem can be done with the IOR software, and you can annotate the output to answer the questions.

I. Consider the problem (*P*) below.
Find graphically all the feasible corner point solutions.

$$(P)\ \max\quad z = 10x_1 + x_2$$
such that
$$x_1 \leq 1$$
$$20x_1 + x_2 \leq 100$$
$$x_1, x_2 \geq 0$$

Then argue verbally that when the usual simplex rules are used, every basic feasible solution will be examined before the optimal solution is reached.¶ Note: no tableaux required!

II. For the (LP) below , check directly that (3,0,2,0) is feasible (satisfies the constraints) with $z = 5$. Using the constraints, create a bound to show that this must be an optimal solution. Identify all the optimal *basic* solutions (*Hint*: each has two basic variables).

$$\text{Maximize } x_1 + x_2 + x_3 + x_4$$
Subject to
$$x_1 + x_2 \leq 3$$
$$x_3 + x_4 \leq 2$$
$$\text{and all } x_i \geq 0$$

III. Consider (P)

$$\begin{array}{llllllll}
\text{Max } z = & 12x_1 & + 9x_2 & + x_3 & - 6x_4 & + 4x_5 & \\
\text{such that} & 3x_1 & + 3x_2 & & & + x_5 & \leq 6 \\
 & x_1 & + 10x_2 & + x_3 & - 9x_4 & - (5/3)x_5 & \leq 2 \\
 & x_1 & + 2x_2 & + (1/3)x_3 & - 2x_4 & & \leq 2
\end{array}$$
$$x_{1,}, x_{2,}, x_3, x_4, x_5 \geq 0$$

The next page shows the sequence of tableaux obtained by following the ordinary simplex rules. After several iterations we are back where we were at a previous iteration. This is a *bona fide* example of cycling! Identify the first place where Bland's Rule would have departed from the given steps. Pick up solving by hand from there (go back to fractions--they are thirds and ninths). What is the optimal solution to (P)? *Hint:* It only takes two more iterations. And for the last, you need only to form the z-row (to test optimality) and the right hand side (to read off the solution).

¶ By generalizing this example, Victor Klee and George Minty (1972) showed that for each n = 2,3,4...., there exists an LP with n decision variables for which the simplex algorithm takes 2^n-1 iterations to find the optimal solution. [V. Klee and G. J. Minty. *How good is the simplex algorithm*? In O. Shisha, editor, *Inequalitites, III*, pages 159-175. Academic Press, New York, 1972.]

Linear Programming Model:

Number of Decision Variables: 5

Number of Functional Constraints: 3

Max Z = 12 X1 + 9 X2 + 1 X3 − 6 X4 + 4 X5

subject to

1) 3 X1 + 3 X2 + 0 X3 + 0 X4 + 1 X5 <= 6

2) 1 X1 + 10 X2 + 1 X3 − 9 X4 −1.667 X5 <= 2

3) 1 X1 + 2 X2 +0.333 X3 − 2 X4 + 0 X5 <= 2

and

X1 >= 0, X2 >= 0, X3 >= 0, X4 >= 0, X5 >= 0.

Solve Interactively by the Simplex Method:

I

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	Right side
Z	0	1	−12	−9	−1	6	−4	0	0	0	0
X6	1	0	3*	3	0	0	1	1	0	0	6
X7	2	0	1	10	1	−9	−1.67	0	1	0	2
X8	3	0	1	2	0.333	−2	0	0	0	1	2

II

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	Right side
Z	0	1	0	3	−1	6	0	4	0	0	24
X1	1	0	1	1	0	0	0.333	0.333	0	0	2
X7	2	0	0	9	1*	−9	−2	−0.33	1	0	0
X8	3	0	0	1	0.333	−2	−0.33	−0.33	0	1	0

III

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	Right side
Z	0	1	0	12	0	-3	-2	3.667	1	0	24
X1	1	0	1	1	0	0	0.333	0.333	0	0	2
X3	2	0	0	9	1	-9	-2	-0.33	1	0	0
X8	3	0	0	-2	0	1*	0.333	-0.22	-0.33	1	0

IV

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	Right side
Z	0	1	0	6	0	0	-1	3	-2e-6	3	24
X1	1	0	1	1	0	0	0.333	0.333	0	0	2
X3	2	0	0	-9	1	0	1*	-2.33	-2	9	0
X4	3	0	0	-2	0	1	0.333	-0.22	-0.33	1	0

V

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	Right side
Z	0	1	0	-3	1	0	0	0.667	-2	12	24
X1	1	0	1	4	-0.33	0	0	1.111	0.667	-3	2
X5	2	0	0	-9	1	0	1	-2.33	-2	9	0
X4	3	0	0	1*	-0.33	1	0	0.556	0.333	-2	0

VI

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	Right side
Z	0	1	0	0	-3e-6	3	0	2.333	-1	6	24
X1	1	0	1	0	1	-4	0	-1.11	-0.67	5	2
X5	2	0	0	0	-2	9	1	2.667	1*	-9	0
X2	3	0	0	1	-0.33	1	0	0.556	0.333	-2	0

The entry -3e-6 is -3/1000000; i.e. essentially 0
The IOR program displays it as a 0.

VII

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	Right side
Z	0	1	0	0	-2	12	1	5	0	-3	24
X1	1	0	1	0	-0.33	2	0.667	0.667	0	-1	2
X7	2	0	0	0	-2	9*	1	2.667	1	-9	0
X2	3	0	0	1	0.333	-2	-0.33	-0.33	0	1	0

VIII

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	Right side
Z	0	1	0	0	0.667	0	-0.33	1.444	-1.33	9	24
X1	1	0	1	0	0.111	0	0.444	0.074	-0.22	1	2
X4	2	0	0	0	-0.22	1	0.111*	0.296	0.111	-1	0
X2	3	0	0	1	-0.11	0	-0.11	0.259	0.222	-1	0

IX

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	Right side
Z	0	1	0	0	-3e-6	3	0	2.333	-1	6	24
X1	1	0	1	0	1	-4	0	-1.11	-0.67	5	2
X5	2	0	0	0	-2	9	1	2.667	1	-9	0
X2	3	0	0	1	-0.33	1	0	0.556	0.333	-2	0

Note: we have arrived back to where we were at Step VI.