MATH 318: Operations Research
Spring 2023

Assignment 3
Due: Monday, March 6

Reading

Read: Hillier & Lieberman, Chapter 3, Sections 5, 6 and 7
Hillier & Lieberman, Chapter 4, Section 1
Olinick, Section II: Convex Sets

Problems (Based on Reading in Olinick)

Fromage Cheese Company Exercises

1. a) Show that the edge from (100,200) to (140,150) can be described analytically as $\{(x, y){:}\ 10x + 8y = 2600,\ 100 \le x \le 140\}$.

b) Along this edge, show that $M = 4.5x + 4y$ has the form $M = 1300 - .5x$.

c) At which vertex of this edge does M achieve its largest value?

2. a) Find an analytic expression for the edge between (140,150) and (200,0),

b) Show that along this edge M can be written as a function of x only.

c) At which vertex of this edge does M take on the largest value?

3. Show that any point on the edge between (100,200) and (140, 150) is an optimal mixture for the revenue function $M = 5x + 4y$. Thus optimal, feasible solutions of LP problems need not be unique.

Feasible, Infeasible and Unbounded Constraint Sets

4. Show that the linear programming problem: Maximize $x + y$ subject to the constraints: $x \ge 0$, $y \ge 0$, $x - y \le -1$, $-x + y \le 0$ has no feasible solutions.

5. Let C be the set of all points (x,y) in the plane satisfying $x \geq 0$, $y \geq 0$, $-x - 2y \leq -8$.

a) Show that C is nonempty and unbounded.

b) Prove that the LP problem: Maximize $M = 2x + 3y$ subject to the constraint that (x, y) lie in C has no feasible, optimal solution.

c) Show that the LP problem: Maximize $M = -3x - 6y$ subject to the constraint that (x, y) lie in C does have an optimal, feasible solution.

6. Let P and Q be any two points in the feasibility set of the Fromage Cheese Company example. Prove that any point on the line segment between these two points also lies in the feasibility set.

Vector Formulation of the LP Problem

7. Prove Theorem 1.

8. Determine A, **b**, and **c** for the linear programming problem:
Maximize $M = 3x + 4y$ subject to the constraints:

$$x \geq 0, \; y \geq 0$$
$$x + 2y \leq 3, \; x + y \leq 2, \; y \leq 8.$$

9. Find the matrix form of the breakfast problem.

Convex Sets

10. Definition: A real-valued function f defined on a convex set K is a *convex function* if $f(\lambda\mathbf{p} + (1 - \lambda)\mathbf{q}) \leq \lambda f(\mathbf{p}) + (1 - \lambda)f(\mathbf{q})$ for all λ between 0 and 1 and all **p** and **q** in K.

a) Prove that every linear function is convex.

b) Let K be the closed interval **[0,1]** and consider the function $f(x) = x^2$. Is f convex? Is f linear?

c) Find some examples of nonconvex functions.