MATH 318: Operations Research
Spring 2023

Assignment 10
Due: Wednesday, May 3

READ: Hillier and Lieberman: Chapter 11

PROBLEMS: Write up clear and complete solutions for the following problems from Hillier and Lieberman: 11.2-3, 11.3-3, 11.3 -5, 11.4 -2

For Problem 11.3-3, Use the following table instead of the one in the text:

		Estimated Grade Points		
Study Days	**Course 1**	**Course 2**	**Course 3**	**Course 4**
1	1	5	4	4
2	3	6	6	4
3	6	8	7	5
4	8	8	9	8

For Problem 11.4-2, use the following table instead of the one in the text:

Investment	**Amount Returned ($)**	**Probability**
A	0 20,000	0.25 0.75
B	10,000 20,000	0.9 0.1

$\boxed{\text{I.}}$ (a) Suppose there are 40 pennies on a table. I begin by removing 1,2,3, or 4 pennies. Then my opponent must remove 1,2,3, or 4 pennies. We continue alternating turns until the last penny is removed. They player who picks up the last match is the loser. Can I be sure of victory? If so, how?

(b) If there are initially *N* pennies and each player removes 1,2,…, or *k* pennies, are there values of *N* and *k* for which I can't be guaranteed a victory? Find such a pair of values or show that I have a winning strategy no matter how *N* and *k* are selected.

$\boxed{\text{II.}}$ Use dynamic programming to solve the following problem

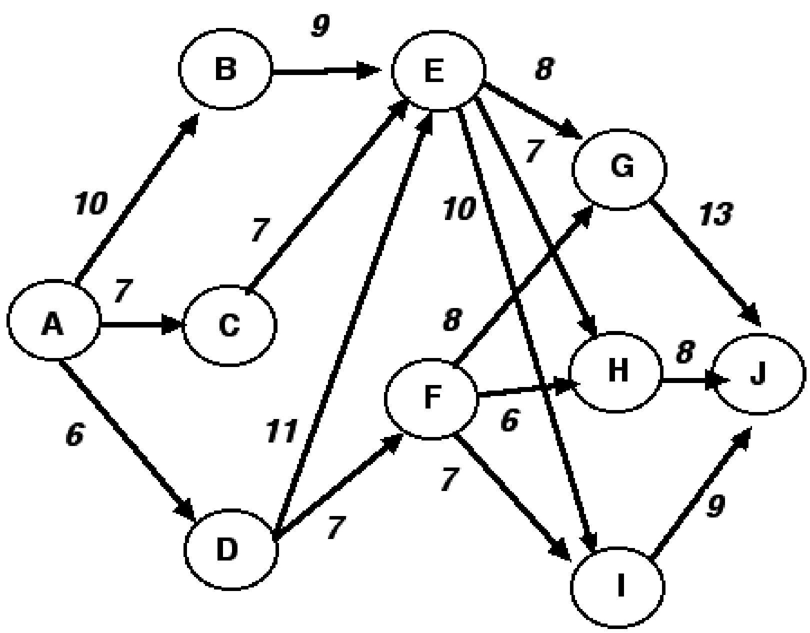
$$\text{Maximize } Z = x_1 x_2^2 x_3^3$$

$$\text{Subject to } x_1 + 2x_2 + 3x_3 \text{ £ } 10$$

and each x_i is a positive integer.

$\boxed{\text{III.}}$ Solve the continuous version of $\boxed{\text{III}}$ that is, replace "$x_i \geq 1$, integer" with "$x_i \geq 0$, real." Solve each stage as a one-variable optimization problem via calculus.

IV. A group of students needs to drive from city *A* to city *J* choosing a route from the network below. Nodes represent cities, and arcs represent roads linking the cities. The number on each arc represents the maximum elevation (in thousands of feet above sea level) when driving between the two cities along the road represented by that arc.



Because the students are towing a heavy trailer, they want to keep the maximum altitude encountered on the trip as small as possible. Find the optimal route by dynamic programming.

VI. Here are the standings in the American League standings on August 30, 1996. Use the maximum flow analysis to determine if (a) Detroit is eliminated and (b) Toronto is eliminated.

Team	Wins	Games To Play	Against New York	Against Baltimore	Against Boston	Against Toronto	Against Detroit
New York	75	28	–	3	8	7	3
Baltimore	71	28	3	–	2	7	4
Boston	69	27	8	2	–	0	0
Toronto	63	27	7	7	0	–	0
Detroit	49	27	3	4	0	0	–