Logistic Models of United States Population Growth

It's easy to cut and paste from text documents or Excel files into Maple. Here we took data for different census years and the corresponding population estimates. Separate the data items with commas and put square brackets at the beginning and end. This creates a vector to which we can assign a name.
Note that the units for the census are millions of people. There have been 24 cenuses since 1790, one every 10 years.

$Years := [1790, 1800, 1810, 1820, 1830, 1840, 1850, 1860, 1870, 1880, 1890, 1900, 1910, 1920, 1930, 1940, 1950, 1960, 1970, 1980, 1990, 2000, 2010, 2020]$

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$Census := [3.9292, 5.3085, 7.2399, 9.6385, 12.8660, 17.0695, 23.1919, 31.4433, 38.5584, 50.1892, 62.9798, 76.2122, 92.2285, 106.0215, 123.2026, 132.1646, 151.3258, 179.3232, 203.2119, 226.5458, 248.7099, 281.4219, 308.7455, 331.4995]$

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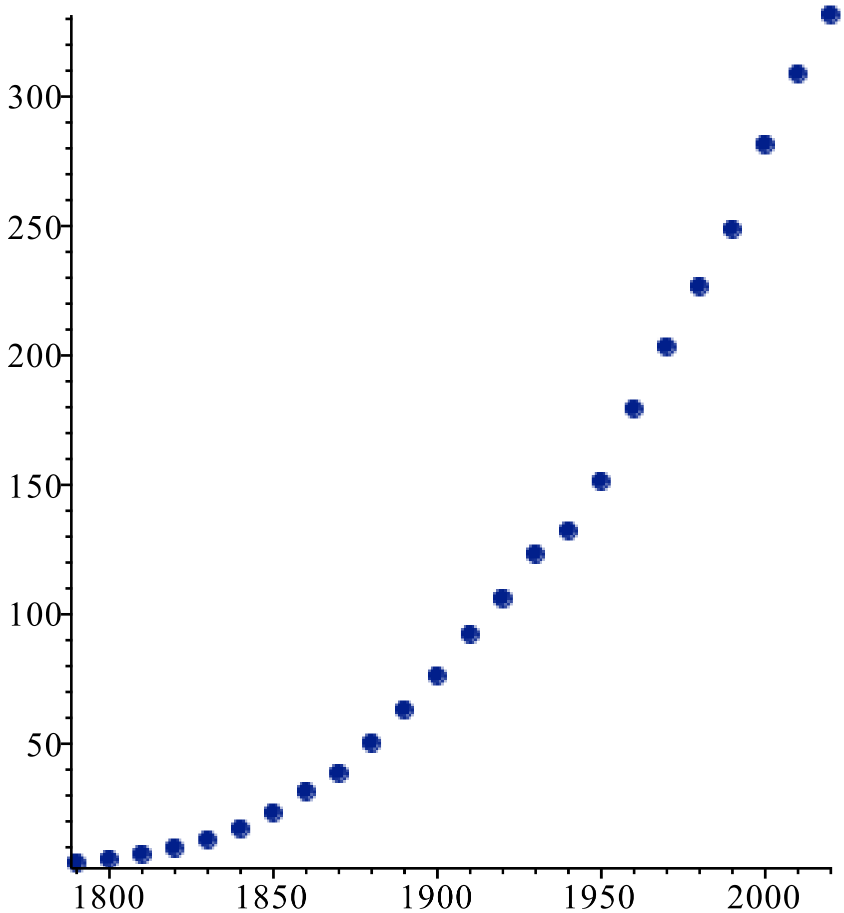
$Plot1 := dataplot(Years, Census, style = point)$



Logistic Growth Model *P'(t) = P(a -b P)* where *k = a/b* is the Carrying Capacity

$Years[3]; Years[13]; Years[23]$

$$1810$$

$$1910$$

$$2010 \tag{3}$$

$Po := Census[3]; P1 := Census[13]; P2 := Census[23]$

$$Po := 7.2399$$

$$P1 := 92.2285$$

$$P2 := 308.7455 \tag{4}$$

$$k := \frac{P1 \cdot (2 \cdot Po \cdot P2 - Po \cdot P1 - P1 \cdot P2)}{Po \cdot P2 - P1^2}$$

$$k := 362.8698819 \tag{5}$$

$A := \ln(k - Po) - \ln(Po)$

$$A := 3.894283420 \tag{6}$$

$B := \ln(k - P1) - \ln(P1)$

$$B := 1.076525438 \tag{7}$$

$C := \ln(k - P2) - \ln(P2)$

$$C := -1.741232547 \tag{8}$$

$a := \left| \frac{(B - A)}{100} \right|$

$$a := 0.02817757982 \tag{9}$$

$d := a \cdot 1810 + A$

$$d := 54.89570289 \tag{10}$$

$P := t \rightarrow \frac{k}{1 + \exp(d - a \cdot t)}$

$$P := t \mapsto \frac{k}{1 + e^{d - a \cdot t}} \tag{11}$$

$P(t)$

$$\frac{362.8698819}{1 + e^{54.89570289 - 0.02817757982\, t}} \tag{12}$$

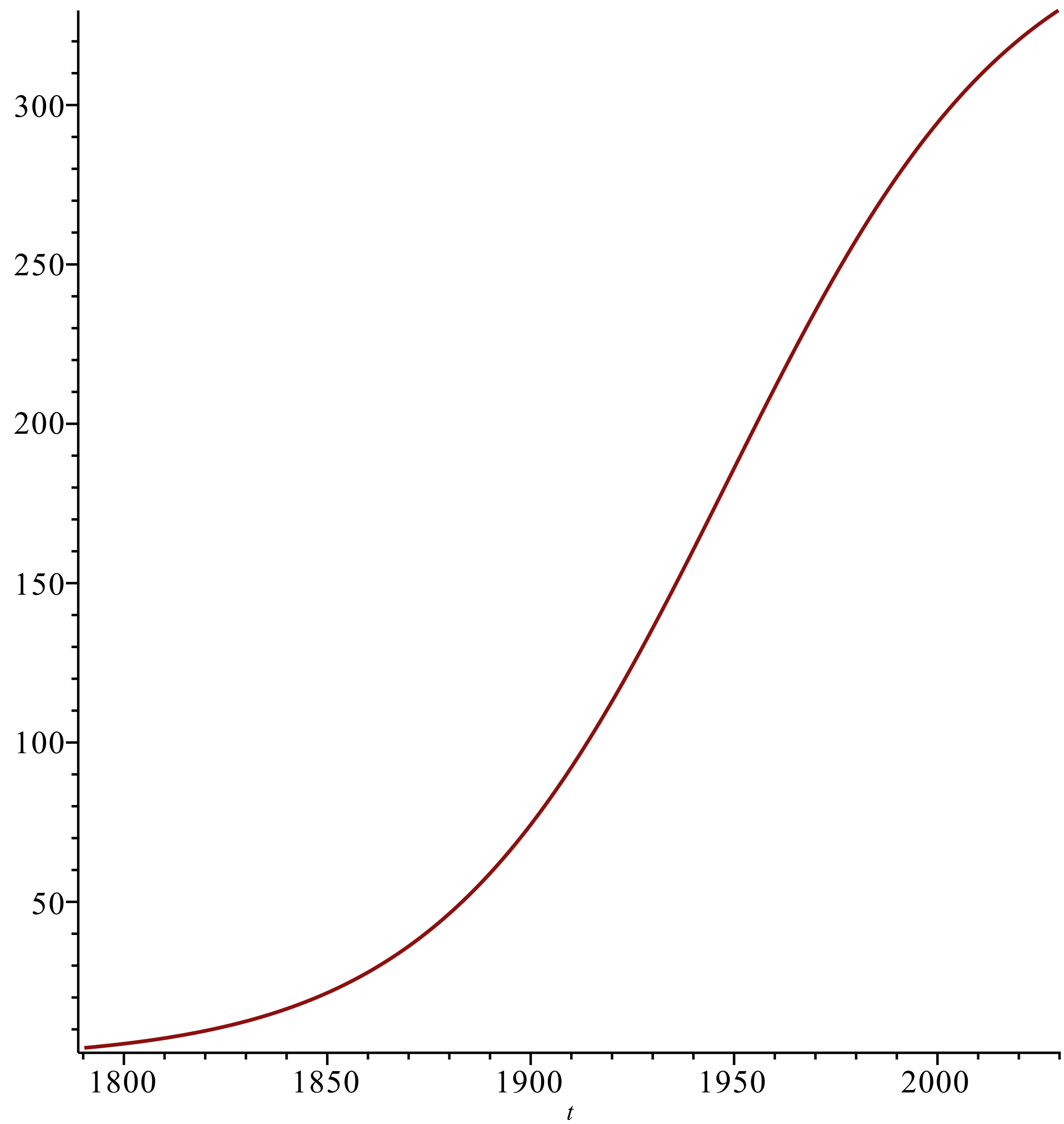
$P(1790); P(2020); P(2023)$

$$4.156562893$$

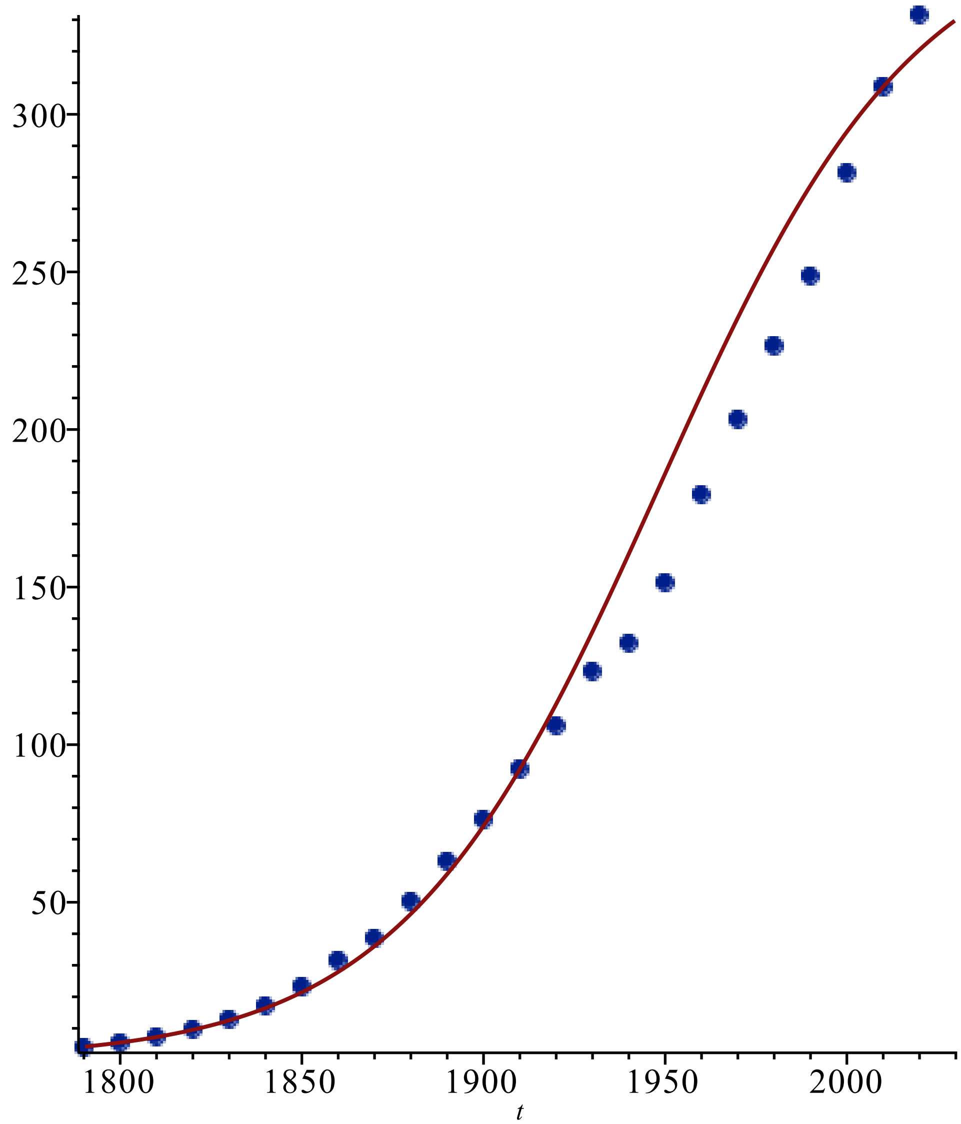
$$320.4836753$$

$$323.5471154 \tag{13}$$

$Plot3 := plot(P(t), t = 1790 ..2030)$



$with(plots) : display(Plot1, Plot3)$



To get a numerical view on the accuracy of the *P* function, we can compute the Error made each year and the Percentage Error; that is, a measure of how large the error is compared to the census number. Note the colon : at the end of each line suppresses the output, but the numbers are stored in these arrays.

```
for i from 1 to 24 do
    Prediction[i] := P(1790 + 10·(i − 1)) :
    Error[i] := Census[i] − Prediction[i] :
    PercentError[i] := Error[i]·100 / Census[i] :
end do:

for i from 0 to 24 do
    if i = 0 then printf("  Year    Census       Predicted      Error     PercentError \n") else
    printf(" %5d   %9.4f   %12.4f   %12.4f  %12.4f   \n", Years[i], Census[i], Prediction[i], Error[i],
      PercentError[i])
```

end if
end do

Year	Census	Predicted	Error	PercentError
1790	3.9292	4.1566	-0.2274	-5.7865
1800	5.3085	5.4890	-0.1805	-3.3999
1810	7.2399	7.2399	0.0000	0.0000
1820	9.6385	9.5344	0.1041	1.0797
1830	12.8660	12.5306	0.3354	2.6072
1840	17.0695	16.4244	0.6451	3.7791
1850	23.1919	21.4542	1.7377	7.4927
1860	31.4433	27.9002	3.5431	11.2681
1870	38.5584	36.0784	2.4800	6.4319
1880	50.1892	46.3222	3.8670	7.7049
1890	62.9798	58.9499	4.0299	6.3988
1900	76.2122	74.2129	1.9993	2.6233
1910	92.2285	92.2285	-0.0000	-0.0000
1920	106.0215	112.9069	-6.8854	-6.4943
1930	123.2026	135.8936	-12.6910	-10.3009
1940	132.1646	160.5542	-28.3896	-21.4805
1950	151.3258	186.0222	-34.6964	-22.9283
1960	179.3232	211.3107	-31.9875	-17.8379
1970	203.2119	235.4598	-32.2479	-15.8691
1980	226.5458	257.6765	-31.1307	-13.7415
1990	248.7099	277.4251	-28.7152	-11.5457
2000	281.4219	294.4506	-13.0287	-4.6296
2010	308.7455	308.7455	-0.0000	-0.0000
2020	331.4995	320.4837	11.0158	3.3230

Sum of Squared Errors

$S := 0$

$$S := 0 \tag{14}$$

for i **from** 1 **to** 24 **do**
  $S := S + (Error[i])^2$
end do:

S

$$6423.729543 \tag{15}$$